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THE INTERPRETATION OF DROPLET DEPOSITION MEASUREMENTS WITH A DIFFUSION MODEL

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Abstract—Measurements of the deposition of 50, 90 and 150 μm droplets on the wall of a 5.08 cm pipe are presented. The droplets were introduced from an orifice in the center of the pipe at approximately the same velocity as downward flowing air. The deposition rate is described by a diffusion model where the diffusion coefficient of the droplets is constant and the velocity profile is uniform. It is shown that special attention has to be paid to the formulation of the boundary condition at the wall.

Key Words: deposition rate, droplets, turbulence, annular flow, dispersed flow, diffusion model

INTRODUCTION

Droplets entrained in air that flows turbulently in a vertical duct move randomly due to their interaction with the turbulent fluid. This paper is concerned with relating the rate of deposition of droplets on a wall to the turbulence properties of the droplets.

The ability of droplets to follow the fluid turbulence is characterized by the reciprocal time constant

$$\beta = \frac{3C_D\rho_f}{4d_p\rho_p}|\mathbf{u}_r|, \qquad [1]$$

where C_D is the drag coefficient, d_p is the particle diameter and $|\mathbf{u}_r|$ is the magnitude of the difference between the radial velocity components of the fluid and the particle. The product of $1/\beta$ and the particle velocity may be thought of as a measure of the stopping distance of this particle if it were moving in a stagnant fluid. For a spherical particle small enough that Stokes' law is applicable,

$$\beta = \frac{18\mu_f}{d_p^2\rho_p}, \qquad [2]$$

where μ_f is the air viscosity.

The focus is on particles which are massive enough that their stopping distance is larger than the thickness of the viscous wall region; i.e. $\tau^+ = u^{*2}/\nu_f\beta$ is > 20. For such particles the non-homogeneity of the turbulence close to the wall is not important and, to a first approximation, the particle turbulence may be considered homogeneous. The applicability of a diffusion model, for which the turbulent diffusivity may be considered independent of its location in a cross-section of the pipe, is explored. The use of a diffusion model implies that the stopping distance is not equal to the pipe radius, so there is also an upper limit on τ^+.

The system studied is the dispersion of droplets downstream of a source that is located at the center of the pipe. The droplets are assumed to collect on the wall of the pipe when they collide with it, so that the pipe wall is a perfect absorber. For situations involving molecular diffusion this would require that the concentration at the wall be zero. However, because the length scale characterizing the droplet motion, L, is so large a finite concentration exists at the wall, in the same way that a Knudsen flow has a finite relative velocity at the wall.

This can be seen by using a radiation boundary condition at the wall,

$$-\epsilon_p \left.\frac{\partial C}{\partial r}\right|_{r=R} = VC(R), \qquad [3]$$

where V is the velocity with which droplets are carried to the wall. If ϵ_p is represented as the product of V and characteristic length L then [3] may be rewritten as

$$VL\left(\frac{\partial C}{\partial r}\right)_{r=R} \cong VC(R).$$

It can be seen that if L is very small (as is the case for molecular diffusion) and $(\partial C/\partial R)_R$ is finite, then $C(R) = 0$. However, if L is of the order of the length scale characterizing $\partial C/\partial r$, then $C(R)$ is a finite number.

A more informative representation of the wall boundary condition is obtained by noting that the mean number of particles per second per unit area that move toward the wall at radius R is given by

$$j_p(R) = C(R)\langle V_r | V_r > 0\rangle f^+, \quad [4]$$

where $\langle V_r | V_r > 0\rangle$ is the conditionally averaged radial velocity, given that the radial velocity is positive, and f^+ is the probability that the radial velocity is positive. The quantity Cf^+ can be interpreted as the number of particles per unit volume having positive radial velocity, and $\langle V_r | V_r > 0\rangle$ is their mean velocity. Therfore, $C(R)$ in [3] should be replaced by $C(R)f^+$ and V by $\langle V_r | V_r > 0\rangle$ if the flux of wall-ward particles in [4] is equated to the flux of deposited particles on the r.h.s. of [3].

Two problems arise in the use of a diffusion model to describe droplet deposition: very large droplets (small β) could be strongly influenced by their method of entry into the field or could have such large turbulence scales that a diffusion mechanism (characterized by a haphazard motion) is inappropriate. A theory to calculate the ratio of the particle and fluid diffusion coefficients, ϵ_p/ϵ_f, and the ratio of the mean-square turbulent velocity fluctuations of the particle and the fluid, $\overline{v_i^2}/\overline{u_i^2}$, has not yet been firmly established.

Because of these difficulties, experiments were initiated in this laboratory in which the turbulence characteristics of the droplets and deposition rates were measured with the same equipment. A continuous stream of droplets of uniform diameter was injected at the same velocity as the gas in order to have well-defined entry conditions. Initial measurements of deposition rates with 50, 90 and 150 μm water droplets have been reported by Lee & Hanratty (1988). These reveal that a dry wall provides a better approximation of a completely absorbing boundary than a wet wall. A recent thesis by Lee (1987) reports on measurements of the turbulence properties of droplets of these sizes obtained with an axially-viewing photographic technique. This paper presents results on the rate of deposition of droplets for the same conditions and experimental equipment used in the turbulence studies of Lee.

The chief motivation for this work is the understanding of the rate of deposition of droplets in a gas/liquid annular flow pattern. This rate process is usually represented by the first-order equation

$$j_p(R) = k_D C_B. \quad [5]$$

Consequently, some discussion is presented at the end of the paper as to what a diffusion model would suggest regarding the dependence of the deposition constant, k_D, on hydrodynamic variables for the experiment discussed in this paper.

The use of a diffusional mechanism to explain particle deposition has been pursued by a number of researchers, starting with Friedlander & Johnstone (1957). Early work was mainly concerned with particles sufficiently small that the stopping distance is less than the thickness of the viscous wall region. The particle and fluid diffusivities were assumed to be equal and the chief differences appear to be in the formulation of the wall boundary condition. Friedlander & Johnstone (1957) assumed that $C = 0$ at a distance from the wall equal to the stopping distance. Davies (1966), Beal (1970) and Sehnel (1970) used the radiation boundary condition [3] at a distance from the wall equal to one stopping distance. Davies assumed the particle and fluid turbulent velocity fluctuations are equal, Beal assumed the particle turbulent velocity fluctuations at one stopping distance are the same as the fluid velocity fluctuations at the edge of the viscous wall region, and Sehnel developed an empirical relation for the particle velocity at one stopping distance.

The works of Hutchinson *et al.* (1971) and Ganic & Mastanaiah (1981) are closer to the interests of the present paper in that they focus on particles for which the stopping distance is larger than the thickness of the viscous wall region. Both developed relations for ϵ_p/ϵ_f. Hutchinson *et al.* assume that $C = 0$ at the wall, while Ganic & Mastanaiah use the radiation boundary condition [3] at the edge of the viscous wall region.

DESCRIPTION OF THE EXPERIMENTS

The deposition results presented in this paper were obtained for the downward flow of air in a 5.08 cm pipe. Droplets of uniform size were injected through an orifice located at the center of the pipe, 60 pipe diameters from the inlet. The test sections were made of brass and were grounded to eliminate any buildup of electrostatic charge. Orifice diameters of 25.4, 50.8 and 76.2 μm were used to produce droplets with diameters of 50, 90 and 150 μm.

Ink was added to the liquid forming the drops during deposition runs that lasted 15–20 min. Droplets adhered to the wall of the pipe and evaporated. At the termination of a run the nine sections of the pipe were disassembled and washed to determine the amount of ink that deposited.

Detailed descriptions of the apparatus and of the techniques used in these deposition studies may be found in previously published articles (Vames & Hanratty 1988; Lee & Hanratty 1988).

INTERPRETATION

The interpretation of the results is simplified by representing the air velocity as a plug flow, by neglecting diffusion in a flow direction and by assuming the particle diffusivity does not depend on r. The differential equation describing the droplet concentration is

$$\frac{\partial C(r,t)}{\partial t} = \varepsilon_p(t)\left[\frac{\partial^2 C(r,t)}{\partial r^2} + \frac{1}{r}\frac{\partial C(r,t)}{\partial r}\right]. \qquad [6]$$

Here, t is the time the particles at a fixed distance from the injector, z, have been in the field.

It is assumed that the thickness of the viscous wall region where the mean velocity is varying is very small compared to the pipe radius so that boundary condition [3] can be applied at the pipe wall. In order to implement this boundary condition it is necessary to relate V to the turbulence properties of the droplets. This is done by making the simplifying assumption that $V = \langle V_r | V_r > 0 \rangle$ is proportional to the r.m.s. velocity $\overline{(v_r^2)}^{1/2}$:

$$V = A\overline{(v_r^2)}^{1/2}. \qquad [7]$$

If, for example, the probability density function of v_r is a joint normal distribution with zero mean, then $A = \sqrt{2/\pi}$.

Boundary condition [3] becomes

$$-\varepsilon_p \frac{\partial C}{\partial r} = A\overline{(v_r^2)}^{1/2} C f^+ \qquad [8]$$

at $r = R$, if f^+ is the fraction of the particles moving toward the wall.

The solution of [6] for a point source at $r = 0$, $t = 0$ is given as

$$C(r,t) = \frac{N}{U_p \pi R^2} \sum_{n=1}^{\infty} \frac{J_0(r\alpha_n)}{J_0^2(R\alpha_n) + J_1^2(R\alpha_n)} \exp\left(-\alpha_n^2 \int_0^t \varepsilon_p \, dt\right), \qquad [9]$$

where N is the total number of droplets injected per unit time, U_p is the mean droplet axial velocity and C is the number of droplets per unit volume. The eigenvalues, α_n, are given by the equation

$$\varepsilon_p \alpha J_1(R\alpha) - \frac{f^+ A}{2} \overline{(v_r^2)}^{1/2} J_0(R\alpha) = 0. \qquad [10]$$

The fraction of the droplets deposited is obtained from [9] using the equation

$$F = \frac{1}{N} \int_0^t \left(-\varepsilon_p \frac{\partial C}{\partial r}\bigg|_R\right) 2\pi R U_p \, dt. \qquad [11]$$

For the experiments reported in this paper the droplets were injected approximately at the fluid velocity, and the free-fall velocity is small compared to the fluid velocity. Therefore, U_p was set equal to U_f. Furthermore, the measurements of the turbulence characteristics of the drops by Lee (1987) and by Vames & Hanratty (1988) suggest that in the region where the deposition is occurring the droplet dispersion has reached its long-time behavior. For example, Vames & Hanratty found a linear $\overline{X^2}$ vs t relation at 5 pipe diameters from the injector. Figure 4 shows that deposition does not start until about 12 pipe diameters. Therefore, the following approximation is made:

$$\int_0^t \varepsilon_p \, dt \cong \varepsilon_{p\infty} t, \qquad [12]$$

where $\varepsilon_{p\infty}$ is the value of ϵ_p at large times.

Figure 1 shows a calculation of the fraction deposited using [9]–[11]. The values of $\varepsilon_p = 13.5\ \text{cm}^2/\text{s}$, $\overline{(v_r^2)}^{1/2} = 46.6\ \text{cm/s}$ and $f^+ = 0.5$, used in these calculations, would be applicable to the runs with 50 μm droplets at Re = 52,000 (Lee 1987). The solid curve in this figure corresponds to the same calculation using $C(R) = 0$ as a boundary condition (B.C.). The use of this boundary condition produces a quite different prediction of the fraction deposited.

Figures 2 and 3 show that the calculated fraction deposited is sensitive to the selection of $\overline{(v_r^2)}^{1/2}$ and ε_p. As would be expected, F increases with increases in these two parameters.

RESULTS

Measurements of the fraction deposited as a function the distance downstream from the injector, expressed as a number of pipe diameters, are given in figure 4. In order to make a comparison with the diffusion model these deposition results are replotted in figure 5 as a function of time for conditions where Lee (1987) obtained measurements of the turbulence characteristics of the drops.

The time, t, was calculated from the equation

$$z = \int_0^t U_p \, dt, \qquad [13]$$

where U_p is obtained by a procedure outlined by Morsi & Alexander (1972). This requires the measurement of the velocity of the particles at time zero and the solution of the equation

$$m_p \frac{dU_p}{dt} = \frac{C_D \rho_f}{2} (U_f - U_p) A_p, \qquad [14]$$

where m_p is the mass of the droplet, A_p is its projected area and C_D is the drag coefficient.

The measurements of $\overline{v_r^2}$ by Lee (1987) are far more accurate than his measurements of ε_p. Therefore, the comparisons with calculations based on [6] and [8] and the assumption of $f^+ = 0.5$,

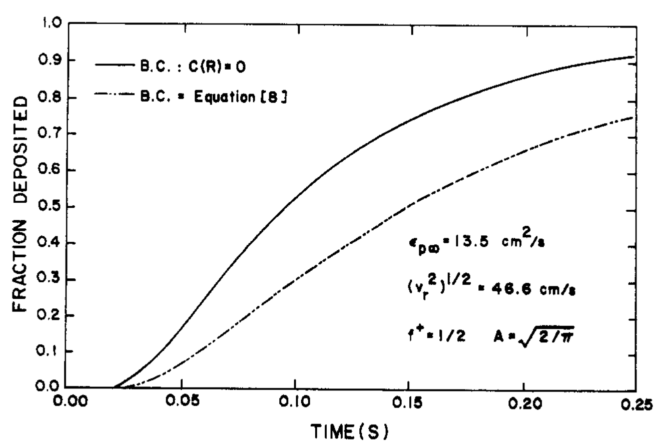

Figure 1. Comparison of predictions of the fraction deposited using the radiation boundary condition [3] and $C(R) = 0$.

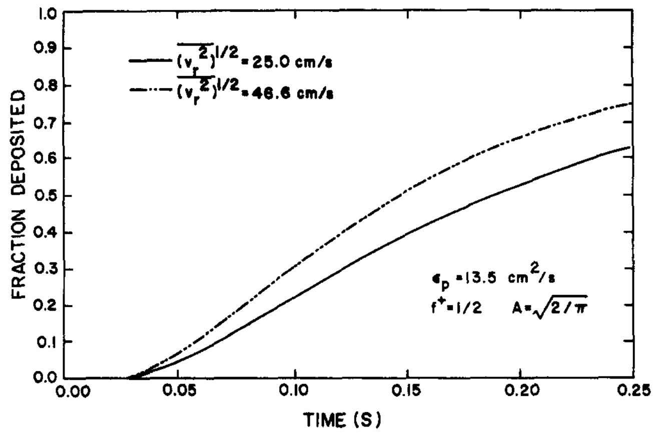

Figure 2. Effect of the droplet r.m.s. radial intensity on the fraction deposited.

shown in figure 5, were done by using measured $\overline{v_r^2}$, given in table 1, and by selecting a value of $\varepsilon_{p\infty}$ that gave the best fit. These $\varepsilon_{p\infty}$ are listed in column 7 of table 1. They are compared (in column 10), with the diffusivity of fluid particles obtained from the equation

$$\frac{\varepsilon_f}{u^*2R} = 0.037, \tag{15}$$

given by Vames & Hanratty (1988) as a best fit to presently available data.

The values of $\varepsilon_p/\varepsilon_f$ for the 50 μm droplets are seen to be approximately equal to or greater than unity. This is consistent with the turbulence measurements presented by Lee (1987) and with the theoretical analysis of Reeks (1977). The values of $\varepsilon_p/\varepsilon_f$ obtained for the 90 μm droplets are just slightly less than unity, while the values of ε_p for the 150 μm droplets are roughly one-half of ε_f. Estimates of ε_p obtained by Lee (1987) from his optical studies of droplet motion are also given in column 8 of table 1. The sample sizes used by Lee were not large enough to obtain accurate measurements of ε_p, so the agreement can be considered satisfactory.

It is of interest to examine the sensitivity of the comparison of the diffusion model to the use of radiation boundary condition [3]. This is done in figure 6. The dashed curve uses the boundary condition [3] while the solid curve uses the boundary condition $C = 0$ at the wall. It is noted that a smaller value of $\varepsilon_{p\infty}$ is required to fit the deposition results when the concentration is assumed zero at the wall. The radiation boundary condition gives a slightly better fit to the measured deposition rates but, more importantly, requires values of $\varepsilon_{p\infty}$ in agreement with point-source diffusion measurements.

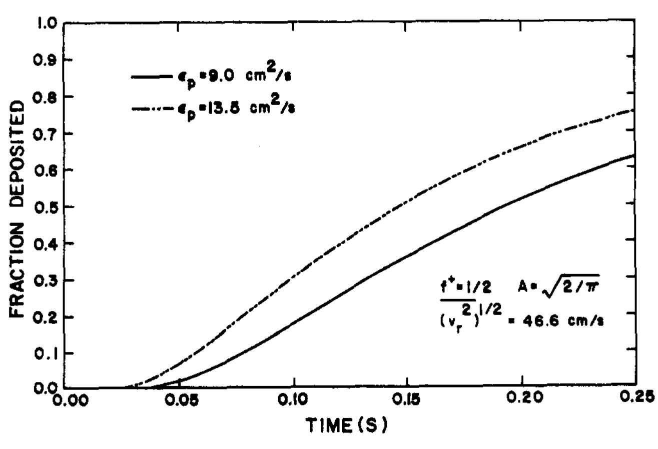

Figure 3. Effect of the droplet eddy diffusivity on the fraction deposited.

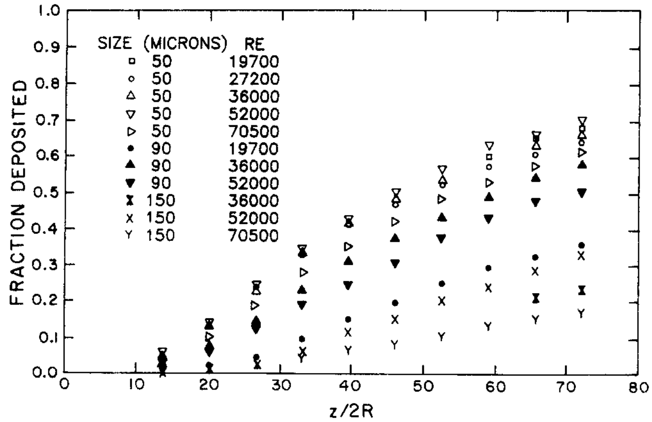

Figure 4. Measurements of the fraction deposited as a function of the distance from the injector.

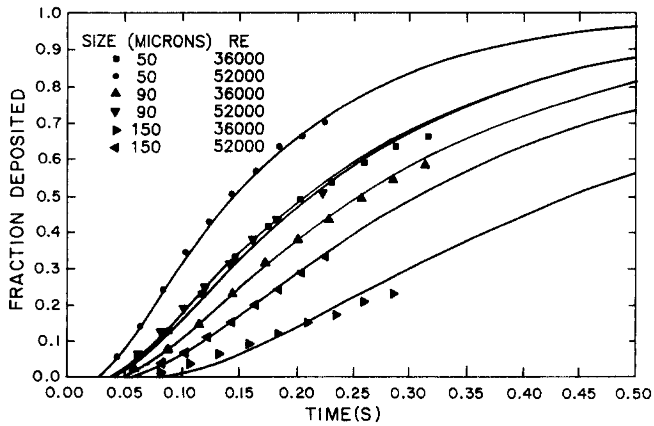

Figure 5. Comparison of the experimental data with calculations based on a diffusion model.

INTERPRETATION OF THE DEPOSITION CONSTANT

As indicated in the introduction, usual practice is to represent deposition rates in terms of a deposition constant, k_D, defined by [5]. It is of interest to examine the implications from diffusional model regarding the interpretation of measurements of k_D presented in this paper.

Since the diffusivity, ε_p, is not varying with radial position one would expect a diffuse concentration profile for a fully-developed condition, similar to what would be found for laminar mass transfer or heat transfer. Thus a mass transfer coefficient, k_l, can be defined by the equation

$$j_p(R) = k_l[C_B - C(R)], \qquad [16]$$

Table 1. Turbulent diffusion coefficient of the droplets

d_p (μm)	$\beta\tau_{Lf}$	Re	u^* (cm/s)	$\overline{(v_r^2)}^{1/2}$ (cm/s)	δ (cm)	$\varepsilon_p{}^a$ (cm²/s)	$\varepsilon_p{}^b$ (cm²/s)	ε_f (cm²/s)	$\dfrac{\varepsilon_p{}^a}{\varepsilon_f}$	$\dfrac{v_T}{\overline{(v_r^2)}^{1/2}}$
50	0.74	36,000	50.5	32.6	0.20	9.2		9.15	1.00	0.21
50	0.61	52,000	70.1	46.6	0.30	13.5		12.6	1.07	0.15
90	0.29	36,000	50.5	23.9	0.44	8.2	8.4	9.15	0.90	0.45
90	0.24	52,000	70.1	27.7	0.50	10.5	13.4	12.6	0.83	0.33
150	0.124	36,000	50.5	18.4	0.78	4.1	5.9	9.15	0.45	1.18
150	0.092	52,000	70.1	21.2	0.94	6.5	8.9	12.6	0.52	0.87

[a]From an empirical fit to the data in figure 5.
[b]Lee (1987).

where C_B is the bulk concentration and $C(R)$ is the concentration close to the pipe wall. A Nusselt number characterizing the diffusion process is defined as

$$\mathrm{Nu} = k_1 \frac{2R}{\varepsilon_p}. \qquad [17]$$

For a fully-developed mass transfer process, Nu would be expected to be roughly constant (=6.8).

A second rate constant, k_2, can be defined by relating $j_p(R)$ to $C(R)$,

$$j_p(R) = k_2 C(R), \qquad [18]$$

where, according to [8],

$$k_2 \cong f^+ A \overline{(v_r^2)}^{1/2} \quad \text{at } R. \qquad [19]$$

The elimination of $C(R)$ from [16] by using [18] and the comparison of the result with [5] yields

$$\frac{1}{k_D} = \frac{1}{k_1} + \frac{1}{k_2}. \qquad [20]$$

Thus the deposition resistance, $1/k_D$, is the sum of resistances $1/k_1$ and $1/k_2$, associated with the diffusion process and the free-flight to the wall. The recognition that k_D may be interpreted in this way could help in explaining the large scatter of measurements of k_D/u^* for large τ^+.

It is of interest to apply this concept to cases for which the relative velocity between the droplets and the fluid is small, $(U_p - U_f)/\overline{(u^2)}^{1/2} < 0.5$. Then, according to the theory of Reeks (1977) and the recent measurements of Vames & Hanratty (1988) and Lee (1987), the particle diffusivity, ε_p, is approximately equal to the fluid diffusivity, ε_f, and the mean-square of the particle turbulent velocity fluctuations can be approximated by

$$\overline{v_r^2} = \overline{u_r^2}\left(\frac{\beta \tau_{Lf}}{0.7 + \beta \tau_{LF}}\right), \qquad [21]$$

where τ_{Lf} is the fluid Lagrangian time constant equal to $\varepsilon_f/\overline{(u_r^2)}$. Turbulence measurements in a pipe [reviewed by Vames & Hanratty (1988)] give

$$\frac{\varepsilon_f}{u^* 2R} = 0.037, \qquad [22]$$

$$\overline{(u_r^2)}^{1/2} \cong 0.9u^* \quad \text{at } r \cong R \qquad [23]$$

and

$$\frac{\tau_{Lf} u^*}{2R} \cong 0.046. \qquad [24]$$

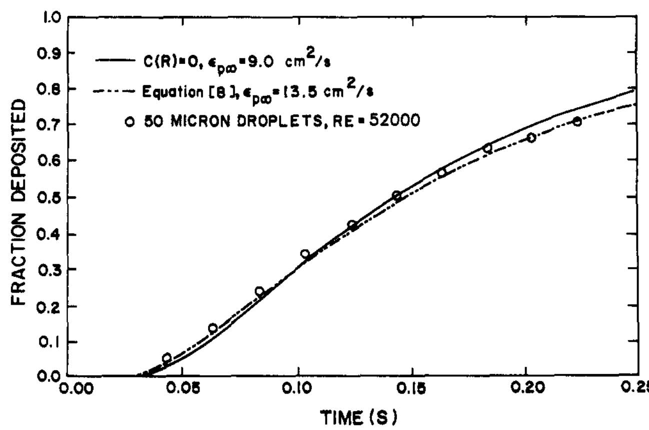

Figure 6. Influence of the boundary condition at the wall on the calculation of the fraction deposited.

By substituting [17], [19] and [22]–[24] into [20] the following relation is obtained:

$$\frac{1}{\dfrac{k_D}{u^*}} = \frac{1}{0.9Af^+\dfrac{(\overline{v_r^2})^{1/2}}{(\overline{u_r^2})^{1/2}}} + \frac{1}{0.037\,\mathrm{Nu}\dfrac{\varepsilon_p}{\varepsilon_f}}. \qquad [25]$$

For the case of $\varepsilon_p/\varepsilon_f \approx 1$, $\mathrm{Nu} = 6.8$, $f^+ = 1/2$, $A = \sqrt{2/\pi}$ and $\overline{v_r^2}$ given by [21], a maximum value of $k_D/u^* = 0.15$ is estimated, and diffusion contributes 59% of the resistance. For this situation of small $(U_p - U_f)/(\overline{u_r^2})^{1/2}$, k_D/u^* will decrease with decreasing τ_{LF} because of decreases in $(\overline{v_r^2})^{1/2}/(\overline{u_r^2})^{1/2}$. For example, if this ratio equals 0.4 then $k_D/u^* = 0.092$, and diffusion contributes 36% of the resistance. For situations in which $(U_p - U_f)/(\overline{u_r^2})^{1/2} > 0.5$, one can expect lower k_D/u^*, since $\varepsilon_p \ll \varepsilon_f$, as a result of the crossing of trajectories.

For a fully-developed dispersion of droplets, rate equation [5] gives the following result for the fraction of droplets at z_0 that are deposited by some distance z if k_D is constant:

$$\ln(1 - F) = -\frac{4k_D}{2RU_f}(z - z_0). \qquad [26]$$

Equation [26] has been found to fit the measurements of F obtained in this study if z_0 is taken as the location where droplets first start to deposit on the wall (see Vames & Hanratty 1988). Values of k_D obtained in this way are represented in column 4 of table 2 and in figure 7. The two solid lines in the figure are correlations suggested by McCoy & Hanratty (1977). The horizontal line is represented by

$$\frac{k_D}{u^*} = 0.17. \qquad [27]$$

It is of interest to note that this is close to the maximum $k_D/u^* = 0.15$ estimated above for a fully-developed dispersed flow.

The upper bound of deposition measurements for annular flows are close to the estimated $k_D/u^* = 0.15$. However, the deposition measurements presented in this paper have an upper bound of $k_D/u^* = 0.10$. This can be explained because the droplet concentration distributions are quite different.

Figure 8 shows Nusselt numbers calculated from a diffusion model for a point source at the pipe center. It is noted that the Nusselt number reaches a fully-developed value of 6.8 at large times. However, the experiments are characterized by smaller contact times for which the Nusselt number varies. The assumption of constant k_D is not correct, so [26] is not strictly valid. The values of k_D presented in column 4 of table 2 are an average characterizing the measured deposition.

Despite this difficulty it is useful to interpret measured k_D/u^* with [25] for fully-developed fields with constant Nusselt number. To do this, $\varepsilon_p/\varepsilon_f$ was assumed equal to unity for the 50 and 90 μm drops and f^+ was taken as 0.5. The 150 μm drops had large relative velocities compared to the turbulence and, from the results in table 1, $\varepsilon_p/\varepsilon_f$ was taken as 0.5. Values of the Nusselt number characterizing the deposition process (shown in column 11 of table 2) were estimated from figure 8. Equations [24] and [21] were used to calculate τ_{Lf} and $\overline{v_r^2}/\overline{u_r^2}$ (columns 8 and 9 of table 2). Approximate agreement is noted between the k_D calculated with [25] and measurements (columns 4 and 14 of table 2). Calculated values of k_1 and k_2 are also listed in columns 13 and 10 of table 2. It is noted that both resistances, $1/k_1$ and $1/k_2$, should be considered.

DISCUSSION

A diffusion model has been shown to describe the deposition of droplets whose stopping distance is greater than the thickness of the viscous wall layer. The importance of using a radiation boundary condition at the wall is demonstrated.

A critical problem in the use of the radiation boundary condition is the specification of the fraction of the droplets moving toward the wall. In all of the calculations presented in this paper

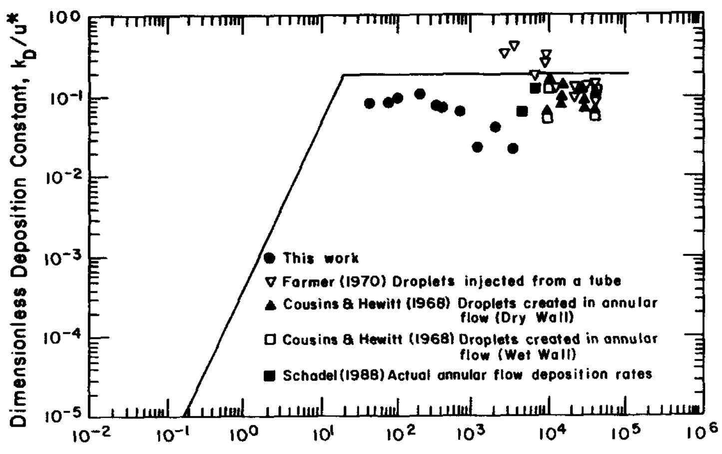

Figure 7. Comparison of k_D/u^* from different experiments.

Table 2. Comparison of measurements of k_D/u^* with calculations based on [25] with $f = 0.5$

d_p (μm)	k_D (cm/s)	u^* (cm/s)	$\frac{k_0}{u^*}$ (meas.)	U_B (cm/s)	β (s^{-1})	$\frac{\beta 2R}{u^*}$	$\beta\tau_f$	$\left(\frac{\overline{v_r^2}}{\overline{u_r^2}}\right)^{1/2}$	$\frac{k_2}{u^*}$	Nu	$\frac{\varepsilon_p}{\varepsilon_f}$	$\frac{k_1}{u^*}$	$\frac{k_D}{u^*}$ (calc.)
50	2.40	29.6	0.08	500	135	23.200	1.060	0.776	0.2790	4.85	1	0.179	0.109
50	2.98	40.0	0.08	700	135	17.140	0.7890	0.728	0.261	4.85	1	0.179	0.106
50	4.30	48.0	0.09	938	135	14.290	0.6570	0.696	0.2500	4.85	1	0.179	0.104
50	6.80	69.8	0.10	1330	135	9.820	0.4520	0.626	0.2240	4.85	1	0.179	0.100
50	7.38	87.2	0.09	1730	135	7.860	0.3620	0.584	0.2090	4.85	1	0.179	0.096
90	3.50	50.8	0.07	924	52	5.200	0.239	0.505	0.1810	4.4	1.0	0.1630	0.086
90	4.21	69.0	0.06	1320	52	3.830	0.1760	0.448	0.1610	4.4	1.0	0.1630	0.081
150	1.25	54.6	0.02	1006	21	1.954	0.0899	0.337	0.1210	2.8	0.5	0.0518	0.036
150	2.44	69.0	0.04	1300	21	1.546	0.0711	0.304	0.1090	2.8	0.5	0.0518	0.035
150	1.52	88.2	0.02	1748	21	1.209	0.056	0.271	0.974	2.8	0.5	0.0518	0.034

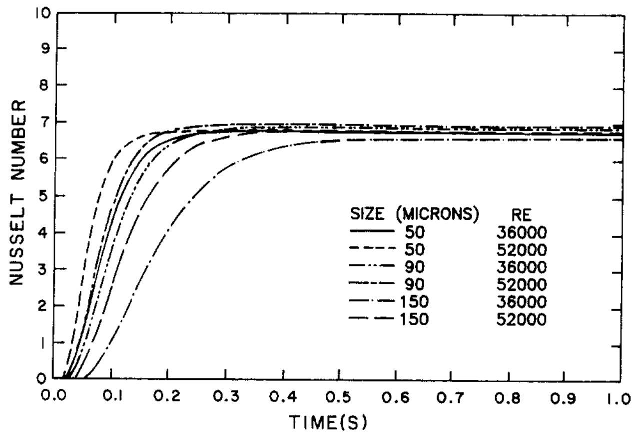

Figure 8. Nusselt number of the droplets originating from a centrally located source as a function of time.

it was assumed that $f^+ = 0.5$ and $Af^+ = 1/\sqrt{2\pi}$. Estimates of f^+ are presented in table 7.3 of the thesis by Lee (1987). These give values of 0.56–0.68 for the 50 μm droplets, 0.70–0.75 for the 90 μm droplets and 0.64–0.70 for the 150 μm droplets. The use of these values of f^+ (rather than 0.5) would require values of ε_p which are 6–15% smaller than those listed in table 1, in order to fit the deposition measurements. A value of $f^+ = 0.5$ was used in the calculations because the sample sizes used by Lee were not large enough to make the estimates of f^+ completely reliable. Lee (1987) also tested the accuracy of calculating V from $\overline{(v_r^2)}^{1/2}$ by using a Gaussian distribution; as is shown in table 7.3 of his thesis this introduces no significant errors.

A more general question about the applicability of the diffusion model is whether the scale of the equipment is large enough compared to the scale characterizing the motion of the drops. It is difficult to give an answer, because it could depend on the method by which the droplets enter the field. However, some criteria can be suggested for the simple system considered in this paper. A Lagrangian scale of particle motion can be defined as $L_p = \varepsilon_p/\overline{(v_r^2)}^{1/2}$. The requirement of a haphazard motion of the particles in the neighborhood of the wall necessitates that L_p/R be small. This can be formulated by the following equation, as can be seen by substituting [22] for ε_f and [23] for $\overline{(u_r^2)}^{1/2}$:

$$\frac{L_p}{R} = 0.082\,\frac{\varepsilon_p}{\varepsilon_f}\,\frac{\overline{(u_r^2)}^{1/2}}{\overline{(v_r^2)}^{1/2}} \ll 1. \qquad [28]$$

For particles with very high inertia (say $\beta\tau_{Lf} \approx 0.02$), L_p/R is close to unity and the particles depositing on the wall are approximated by a free-flight from the center.

McCoy & Hanratty (1979) have treated the limiting behavior of unidirectional dispersion for which $f^+ = 1$. If the distribution function describing $\overline{X_p^2}(t)$ is Gaussian and if (again) a plug flow approximation is made, the concentration of particles at the wall is given by

$$C(R) = \frac{N}{2\overline{X_p^2}\,U_p}\exp\left(-\frac{R^2}{2\overline{X_p^2}}\right), \qquad [29]$$

with

$$\overline{X_p^2} = \overline{v_r^2}\,t^2 = \frac{\overline{v_r^2}}{U_p^2}\,z^2. \qquad [30]$$

The fraction of droplets deposited over a distance z from the injector is

$$F = \exp\left(-\frac{R^2}{2\overline{X_p^2}}\right), \qquad [31]$$

where $\overline{X_p^2}$ is given by [30]. In this limit the Gaussian concentration profiles downstream of a point source reflect the distribution of particle velocities. Because of this the droplets with larger velocities deposit first and calculated values of $k_2 = R_D/C(R)$ given by McCoy & Hanratty decrease with increasing z.

It is not clear how to treat cases which are intermediate between the diffusion model and unidirectional dispersion. It is possible that the measurements with the 150 μm droplets fall in this category. The smaller values of $\varepsilon_p/\varepsilon_f$ used to characterize their deposition could result from an inadequate time to reach a fully haphazard motion, as much as to the crossing of trajectories phenomenon.

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